

Holographic Vector Dominance for the Nucleon

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We derive a two-parameter formula for the electromagnetic form factors of the nucleon described as an instanton by “integrating out” all KK modes other than the lowest mesons from the infinite-tower of vector mesons in holographic QCD while preserving hidden local symmetry for the resultant vector fields. With only two parameters, the proton Sachs form factors can be fit surprisingly well to the available experimental data for momentum transfers $Q^2 \lesssim 0.5 \text{ GeV}^2$ with $\chi^2/\text{dof} \lesssim 2$, indicating the importance of an infinite tower in the soliton structure of the nucleon. The prediction of the Sakai-Sugimoto holographic dual model is checked against the fit values that we interpret as representing Nature. The nature of the “core” in the nucleon structure is reassessed in light of the results obtained with the infinite-tower structure.

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The celebrated Sakurai vector dominance (sVD for short) model for the EM form factors [1] works fairly well for mesons [2] at zero temperature and zero density but it famously fails for the nucleon [3–5]. The failure has been interpreted as an indication that the nucleon has a “core” which is not present in the pion structure [3, 4]. The “core” has been attributed – among a variety of sources – to a compact microscopic structure of QCD variables, such as for instance a little chiral bag with quarks confined within [4].

The recent development of holographic dual QCD (hQCD for short), specially, the Sakai-Sugimoto string theory model [6] that implements correctly chiral symmetry of QCD, indicates a dramatic return of the notion of vector dominance for *both* mesons and baryons [7–10]. What characterizes the baryon structure in the hQCD model is that the baryon emerges as a soliton in the presence of an infinite tower of vector mesons in addition to the pion. It is the infinite tower that renders the VD description applicable to both mesons and baryons. In fact, the isovector component of the EM form factor has the “universal” form

$$F_V^h = \sum_n \frac{g_{v^n} g_{v^n h h}}{Q^2 + m_{v^n}^2} \quad (1)$$

where g_{v^n} is the photon- v^n coupling and $g_{v^n h h}$ is the v^n - $h h$ coupling for $h = \pi, N$. Here v^n is the n -th isovector vector mesons ρ, ρ', \dots in the tower. What distinguishes one from the other is the vector-meson coupling to the hadron. This form – which turns out to be almost completely saturated by the first four vector mesons – works surprisingly well for both mesons and baryons at low-momentum transfers, say, $Q^2 \lesssim 0.5 \text{ GeV}^2$.

Three questions arise from these results.

The first is what makes the difference between the “good” sVD for the pion and the “bad” sVD for the nucleon disappear when the infinite tower is present?

The second is the problem of the “core.” While it is reasonable to view the pion as point-like when probed at

long wavelength, the nucleon is an extended object, for which a local field description must break down at some – not too high – momentum scale. There are indications from high-energy proton-proton scattering, experiments on high mass muon pairs and also in deep inelastic scattering off nucleon that the nucleon has a core of $\sim 0.2 - 0.3 \text{ fm}$ in size [11]. It is this class of observations that led to the notion of the “Little Bag” [12] and to the hybrid structure of the nucleon with a core made up of a quark bag surrounded by a meson cloud [4]. The question is whether the core is subsumed in the infinite tower and if so, where and in what form. Closely tied to this question is: Does the photon “see” the size of the instanton – which is a skyrmion in the infinite tower of vector mesons – which goes as $\sim 1/\sqrt{\lambda}$ where $\lambda = N_c g_{YM}^2$ is the 't Hooft constant? For a large value of λ , the soliton(instanton) size is small. However this size cannot be a physical quantity. The physical size should be independent of λ , which is akin to the bag size which is also unphysical according to the Cheshire Cat principle [10]. So the last question is: Is the “core” a physical observable?

The objective of this Letter is to address the above three questions.

We first define the notations that we shall use and then give a concise summary of the hQCD calculation of the form factors as described in [7–9]. We shall follow the notations of [7, 8].

The nucleon form factors are defined from the matrix elements of the external currents,

$$\langle p' | J^\mu(x) | p \rangle = e^{iqx} \bar{u}(p') \mathcal{O}^\mu(p, p') u(p), \quad (2)$$

where $q = p' - p$ and $u(p)$ is the nucleon spinor of momentum p . From the Lorentz invariance and the current conservation, with the assumption of CP invariance, the

operator \mathcal{O}^μ takes the form (say, for the EM current)

$$\mathcal{O}^\mu(p, p') = \gamma^\mu \left[\frac{1}{2} F_1^s(Q^2) + F_1^v(Q^2) \tau^3 \right] + i \frac{\sigma^{\mu\nu}}{2m_N} q_\nu \left[\frac{1}{2} F_2^s(Q^2) + F_2^v(Q^2) \tau^3 \right], \quad (3)$$

where m_N is the nucleon mass and $\tau^a = \sigma^a/2$. $F_1^{s,v}$ and $F_2^{s,v}$ are the Dirac and Pauli form factors for isoscalar (s) current and isovector (v) respectively.

As matrix elements, the form factors contain all one-particle irreducible diagrams for two nucleons and one external current. Thus they are very difficult to calculate in QCD. It turns out, however, that the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, or gravity/gauge theory correspondence, found in certain types of string theory, enables one to compute such non-perturbative quantities as hadron form factors within certain approximations.

According to the AdS/CFT correspondence, the low energy effective action of the gravity dual in the bulk sector of QCD becomes the generating functional for the correlators in the gauge sector of operator \mathcal{O} in QCD in the large N_c limit. The normalizable modes of the bulk field are identified as the physical states in QCD, created by the operator \mathcal{O} .

The model we shall use is the gravity dual of low energy QCD with massless flavors in the large N_c (or quenched) approximation constructed by Sakai and Sugimoto (SS) [6]. The holographic dual of spin- $\frac{1}{2}$ baryons, or nucleons, for two flavors that we shall adopt in this paper was constructed in [7] by introducing a bulk baryon field, whose effective action is given in the ‘‘conformal coordinate’’ (x, w) as

$$S_{5D}^{\text{eff}} = \int_{x,w} (-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - im_b(w)\bar{\mathcal{B}}\mathcal{B} + \kappa(w)\bar{\mathcal{B}}\gamma^{mn} F_{mn}^{SU(2)_I} \mathcal{B} + \dots) + S_{\text{meson}}, \quad (4)$$

where \mathcal{B} is the 5D bulk baryon field, D_m is the gauge covariant derivative and S_{meson} is the effective action for the mesons. Using the instanton nature of baryon, the coefficients $m_b(w)$ and $\kappa(w)$ can be reliably calculated in string theory. In (4) the ellipsis stands for higher derivative terms that are expected to be suppressed at low energy, $E < M_{KK}$, where the KK mass sets the cut-off scale. Note that the magnetic coupling involves only the non-abelian part of the flavor symmetry $SU(2)_I$ – with abelian $U(1)_B$ being absent – due to the non-abelian nature of instanton-baryons.

The (nonnormalizable) photon field is written as

$$A_\mu(x, w) = \int_q A_\mu(q) A(q, w) e^{iqx}, \quad (5)$$

with boundary conditions that $A(q, w) = 1$ and $\partial_w A(q, w) = 0$ at the UV boundary, $w = \pm w_{\text{max}}$ and the (normalizable) bulk baryon field as

$$\mathcal{B}(w, x) = \int_p [f_L(w) u_L(p) + f_R(w) u_R(p)] e^{ipx}. \quad (6)$$

These 5D wave functions, $A(q, w)$ and $f_{L,R}(w)$, are determined by solving the equation of motion from our action (4). Then, using the AdS/CFT correspondence, one can read off the Dirac form factor and the Pauli form factor [7, 9],

$$F_1^p(Q^2) = \sum_{k=0}^{\infty} \frac{g_V^{(k)} g_{v^k}}{Q^2 + m_{2k+1}^2}, F_2^p(Q^2) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{g_2^{(k)} g_{v^k}}{Q^2 + m_{2k+1}^2} \quad (7)$$

with

$$g_V^{(k)} = \int_{-w_{\text{max}}}^{w_{\text{max}}} dw |f_L(w)|^2 \times (\psi_{(2k+1)}(w) + \kappa(w) \partial_w \psi_{(2k+1)}(w)) \quad (8)$$

$$g_2^{(k)} / 4m_N = \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \kappa(w) f_L^*(w) f_R(w) \psi_{(2k+1)}(w) \quad (9)$$

where $g_{v^k} = \zeta_k m_{2k+1}^2$ is the v^k decay constant and $\psi_{(2k+1)}$ is the wave function of the vector meson given in [6]. (Here $g_V^{(k)}$ is $g_{v^k NN}$ of (1)). This is the set of formulas that we shall use for our analysis that follows.

For comparison with experiments, it is more convenient to work with the Sachs form factors. They are given in terms of the Dirac and Pauli form factors defined above as

$$G_E^p(Q^2) = F_1^p(Q^2) - \frac{Q^2}{4m_N^2} F_2^p(Q^2), \quad (10)$$

$$G_M^p(Q^2) = F_1^p(Q^2) + F_2^p(Q^2). \quad (11)$$

In order to focus on the role that the lowest vector mesons $V^{(0)} \equiv (\rho, \omega)$ play in the form factors and expose the importance of the tower, we would like to integrate out all other vector mesons than the $V^{(0)}$. In doing this, it is important to preserve hidden local symmetry for $V^{(0)}$ as well as chiral symmetry. How this can be done has been worked out for the pion [13, 14].

Following the strategy proposed in [14] for the pion form factor, we first obtain an effective 4D Lagrangian for an infinite tower of KK modes and the nucleon from the 5D one in (4) by performing the integration over w . Then, we integrate out all KK modes other than $V^{(0)}$ through the equations of motion for the higher KK modes. This procedure gives an effective 4D Lagrangian to $\mathcal{O}(p^4)$ for $V^{(0)}$ and the nucleon invariant under the hidden local symmetry as well as chiral symmetry. Going to higher orders cannot be justified unless one incorporates higher order terms in the DBI action and loop corrections in the bulk sector which is not at present doable. What we are doing is essentially to ‘‘integrate out’’ the tower at the tree level with the effects of the higher tower lodged in the action given to $\mathcal{O}(p^4)$. It is in this sense that we are exposing the infinite tower effect as *corrections* to the lowest KK mode. The validity of such a procedure is clearly limited to low momentum transfer. We are thus limiting to $Q^2 \lesssim 0.5 \text{ GeV}^2$.

An important point to note here is that the formula we derive is quite generic for low energy processes cut off at the KK mass (M_{KK}) scale (or equivalently at the chiral scale $\Lambda_\chi \sim 4\pi f_\pi$). For the generic formula (12) that is deduced from (1), what matters is the infinite tower structure, which could be “dimensionally deconstructed” bottom-up [15] or reduced à la Klein-Kaluza from string theory top-down.

We shall show explicitly in an extended version how the reasoning made in [14] for the pion form factor can be taken over for the nucleon case. It is however not difficult to see intuitively what happens. As shown in [7], the crux of the matter is that the direct photon coupling to the instanton can be eliminated by a field redefinition of the photon field and the charge sum rule, so the form factor is entirely vector dominated by the infinite tower just as the pion form factor is. The only difference is in $g_{v^n h h}$, namely the coupling of the vector meson v^n to hadron $h = \pi, N$. The instantonic structure of the baryon is then buried in the coupling $g_{v^n N N}$. It can be shown that when all the KK modes other than the lowest one $V^{(0)}$ are integrated out, the resultant Sachs form factors take the form

$$G_{E,M}^p(Q^2)/\beta_{E,M} = \left(1 - \frac{a_{E,M}}{2}\right) + z_{E,M} \frac{Q^2}{m_\rho^2} + \frac{a_{E,M}}{2} \frac{m_\rho^2}{m_\rho^2 + Q^2}, \quad (12)$$

where $\beta_E = 1$ and $\beta_M = \mu_p$, and $a_{E,M}$ and $z_{E,M}$ are parameters to be determined. It follows from arguments in complete parallel to those given in [14] for the pion form factor that formally integrating out higher modes is equivalent to expanding (7) in Q^2/m_{2k+1}^2 for $k \geq 1$ up to $\mathcal{O}(Q^2)$ while keeping the ρ meson ($k=0$ mode) propagator as it is in consistency with perturbative unitarity.

In what follows we do this expansion explicitly for the electric form factor, but the same holds for the magnetic form factor. The result for the electric form factor is

$$G_E^p(Q^2) = g_V^{(0)} \frac{m_0^2}{Q^2 + m_0^2} + \left[1 - g_V^{(0)}\right] - \frac{Q^2}{m_0^2} \sum_{k=1}^{\infty} g_V^{(k)} \frac{\zeta_k m_0^2}{m_{2k+1}^2} - \frac{Q^2}{8m_N^2} g_2 \quad (13)$$

where we have used the sum rules $\sum_{k=0}^{\infty} g_V^{(k)} \zeta_k = 1$ and $\sum_{k=0}^{\infty} g_2^{(k)} \zeta_k = g_2$. Comparing (12) and (13), we find the a and z parameters for the electric form factor

$$a_E^{(\text{hQCD})} = 2g_V^{(0)} \zeta_0, \quad (14)$$

$$z_E^{(\text{hQCD})} = - \sum_{k=1}^{\infty} g_V^{(k)} \frac{\zeta_k m_0^2}{m_{2k+1}^2} - \frac{m_0^2}{8m_N^2} g_2. \quad (15)$$

Making a similar expansion for the magnetic form fac-

tor, one finds from (7)

$$a_M^{(\text{hQCD})} = \left[2g_V^{(0)} \zeta_0 + g_2^{(0)} \zeta_0\right] / \mu_p, \quad (16)$$

$$z_M^{(\text{hQCD})} = - \sum_{k=1}^{\infty} \left[g_V^{(k)} + \frac{1}{2}g_2^{(k)}\right] \frac{m_0^2 \zeta_k}{m_{2k+1}^2} / \mu_p, \quad (17)$$

with $\mu_p = 1 + (1/2)g_2$.

We now turn to the analysis of Eq.(12) done in three different ways. It is in the a and z parameters that hadron structure figures.

First we consider the parameters (a, z) as *totally free* and best-fit (12) to the accurate experiments given in Ref. [16]. Given that the approximation is valid for low momentum transfers, we limit to $Q^2 \leq 0.5 \text{ GeV}^2$. Throughout we use the values

$$m_\rho = 0.775 \text{ GeV}, m_N = 0.938 \text{ GeV}. \quad (18)$$

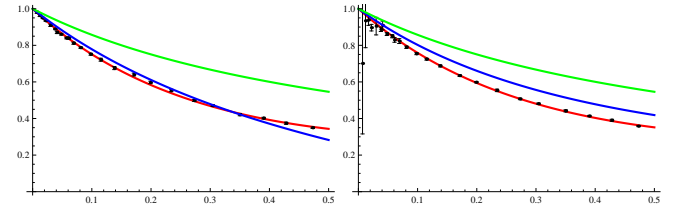


FIG. 1: (color online) G_E^p (left panel) and G_M^p/μ_p (right panel) vs. Q^2 . The horizontal axis is Q^2 in units of GeV^2 . The red, green and blue curves are, respectively, the “best fit,” the sVD prediction and the hVD prediction.

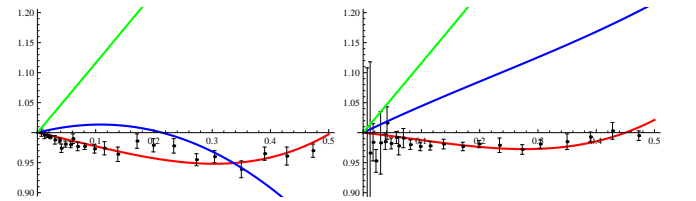


FIG. 2: (color online) G_E^p/G_D (left panel) and $G_E^p/\mu_p G_D$ (right panel) vs. Q^2 . See Fig. 1 for labels.

The best χ^2 fit is given in red in Fig 1 for the form factors and in Fig 2 for the same divided by the dipole form factor $G_D(Q^2) = \left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^{-2}$ which has been proven to be fairly close to Nature. The best fit parameters and χ^2 's come out to be

$$G_E^p : a_E^{(\text{best})} = 4.55, z_E^{(\text{best})} = 0.45; \chi_E^2/\text{dof} = 1.5(19)$$

$$G_M^p : a_M^{(\text{best})} = 4.31, z_M^{(\text{best})} = 0.40; \chi_M^2/\text{dof} = 1.1(20)$$

Since the fit parameters are close to each other with similar χ^2 , we may assume that $a_E = a_M$ and $z_E = z_M$ although we have no compelling reason to do so, and

make the best-fit. The result is: $a^{(best)} = 4.42$ and $z^{(best)} = 0.42$ with $\chi^2/\text{dof} = 1.90$, quite close to (19) and (20). It is interesting to compare the best-fit for the nucleon to the best-fit for the pion form factor as obtained in [14]: $a_\pi^{(best)} = 2.44$ and $z_\pi^{(best)} = 0.08$ with $\chi^2/\text{dof}=2.44$ while the sVD with $a = 2$ and $z = 0$ gives $\chi^2/\text{dof}=4.3$. Note that in the case of the pion, the deviation in χ^2 of the best-fit from the sVD is relatively small accounting for the general acceptance of the sVD.

We now examine how the vector dominance models, sVD and hVD, fare in predicting these parameters and in fitting the data for the nucleon.

First we consider the sVD. It has been known that the sVD does not work at all for the nucleon although it has been fairly successful for mesons. The sVD corresponds to taking in (12)

$$a_E^{VD} = 2, z_E^{VD} = -m_\rho^2/(4m_N^2) \simeq -0.171, \quad (21)$$

$$a_M^{VD} = 2, z_M^{VD} = 0. \quad (22)$$

(The second term in (21) comes from the second term of (10)). The result is shown in green in Figs. 1 and 2. The χ^2/dof comes out to be 187 and 852, respectively, for G_E^p and for G_M^p . This result merely confirms the well-known story that sVD does not work for the nucleon.

Now turning to hVD, we will use the results of [7]. We are limiting to the lowest four states since it is found numerically that the charge and magnetic sum rules are almost completely saturated by them [8]. One should however be careful in using this observation for form factors since the four states may not saturate momentum-dependent observables as fully as the static quantities.

By using the values listed in Table 2 of the first reference of [7], the parameter a comes out to be

$$a_E^{(\text{hQCD})} = 3.01, a_M^{(\text{hQCD})} = 3.14, \quad (23)$$

As for $z_{E,M}$, there are no known sum rules for the sums in Eqs. (15) and (17). We shall simply take the values for $k = 1, 2, 3$ from the table

$$z_E^{(\text{hQCD})} \simeq -\sum_{k=1}^3 g_V^{(k)} \frac{\zeta_k m_0^2}{m_{2k+1}^2} - \frac{m_0^2}{8m_N^2} g_2 = -0.0424 \quad (24)$$

$$\mu_p z_M^{(\text{hQCD})} \simeq -\sum_{k=1}^3 g_V^{(k)} \zeta_k + \frac{1}{2} g_2^{(k)} \zeta_k \frac{m_0^2}{m_{2k+1}^2} = 0.1625 \quad (25)$$

The form factors predicted in hQCD using the constants so determined, i.e., $(a_E, z_E) = (3.01, -0.042)$, $(a_M, z_M) = (3.14, 0.16)$, are plotted in blue in Figs. 1 and 2. The χ^2 comes out to be 20.2 for G_E^p and 133 for G_M^p . While G_E comes out to be reasonable, G_M is not.

To conclude, while the infinite tower structure of hQCD model gives the pion form factor that deviates little from the Sakurai VD, the nucleon form factor deviates drastically from it. In the way the form factor is parameterized, the $V^{(0)} = \rho, \omega$ contribution to the nucleon form factor is considerably greater than that to the pion, so the conventional VD with the “universality” does not hold at all. However this feature suggested by Nature is reproduced semi-quantitatively in the hQCD model of Sakai and Sugimoto. For instance, the charge contributed from the $V^{(0)}$ overshoots the proton charge by more than 40%, which then is dominantly compensated by the negative charge contribution coming from the first excited vector mesons. This requires the so-called “core” contribution to be large but of the sign opposite to what has been associated with a possible short-distance QCD component [3–5]. This result brings us to question the *standard* interpretation of the experimental observation [11] as an evidence for a microscopic QCD degree of freedom.

The deviation from the fit values of the parameters (a, z) in the hQCD prediction for the nucleon, particularly in the magnetic form factor, may be pointing to the potential importance of what is manifestly missing in the hQCD model, namely, $1/N_c$ corrections, quark-mass effects and/or certain curvature effects in the bulk sector. This is an open issue in the field.

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